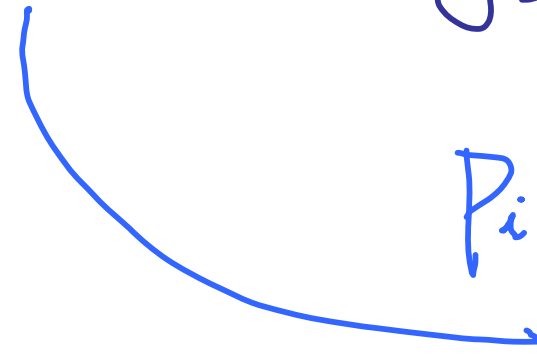




ENTROPY

$$I(s_i) = \log_2\left(\frac{1}{p_i}\right) \text{ bits}$$

$$p_i = \Pr(s_i)$$


$$I(s_i) \xrightarrow{\text{as } p_i \rightarrow 0} \infty$$

$$I(s_i) \xrightarrow{\text{as } p_i \rightarrow 1} 0$$

Average Information
of source S is

$$H(S) = E\{I(S)\} \\ = \sum_{i=0}^{M-1} P_i I(s_i)$$

$$H(S) = \sum_{i=0}^{M-1} P_i \log_2 \left(\frac{1}{P_i} \right)$$

Entropy of source S

weighted $I(s_i)$ by probability P_i

↓
Entropy: Characterizes the
average Information
of source.

↙
Average
Uncertainty associ-
ated with source.

↘
average
information per
symbol emitted/
generated by
source.

More uncertainty
⇒ More information

$$H(S) = \sum_{i=0}^{M-1} P_i \log_2 \left(\frac{1}{P_i} \right)$$

$$0 \leq P_i \leq 1$$

$$\Rightarrow 1 \leq \frac{1}{P_i} < \infty$$

$$\Rightarrow \boxed{\log_2 \left(\frac{1}{P_i} \right) \geq 0}$$

$$\Rightarrow P_i \log_2 \left(\frac{1}{P_i} \right) \geq 0 \quad \text{for each } i$$

sum of
non-negative
terms

$$\Downarrow$$

$$H(S) \geq 0$$

Entropy is non-negative

$$\underline{H(S) \geq 0}$$

Information content
of source ≥ 0 .

Further, if $P_i = 1$

$$\frac{P_i \log_2 \left(\frac{1}{P_i} \right)}{0} = \frac{1 \cdot \log_2 \left(\frac{1}{1} \right)}{0}$$
$$= 0.$$

$= 0$ for $P_i = 1$

$P_i = 0$?

$$P_i \log_2 \left(\frac{1}{P_i} \right) = \frac{\log_2 \left(\frac{1}{P_i} \right)}{\frac{1}{P_i}}$$

✓
Numerator
& Denominator
→ ∞ as $P_i \rightarrow 0$

$$\lim_{P_i \rightarrow 0} \frac{\log_2\left(\frac{1}{P_i}\right)}{\left(\frac{1}{P_i}\right)}$$

$$= \lim_{P_i \rightarrow 0} \frac{-\log_2 P_i}{\frac{1}{P_i}}$$

L'Hopital
Rule
↓

$$= \lim_{P_i \rightarrow 0} \frac{\frac{d}{dP_i} (-\log_2 P_i)}{\frac{d}{dP_i} \left(\frac{1}{P_i}\right)}$$

$$\begin{aligned}
 &= \lim_{P_i \rightarrow 0} \frac{\frac{d}{dP_i}(-\log_e P_i \log_2 e)}{-\frac{1}{P_i^2}} \\
 &= \lim_{P_i \rightarrow 0} \left(-\frac{1}{P_i}\right) \cdot \log_2 e (-P_i^2) \\
 &= \lim_{P_i \rightarrow 0} P_i \log_2 e = 0.
 \end{aligned}$$

$$\lim_{P_i \rightarrow 0} P_i \log_2 \left(\frac{1}{P_i}\right) \rightarrow 0$$

$$= 1) \quad P_i \log_2 \left(\frac{1}{P_i}\right) = 0 \text{ for } P_i = 1$$

$$P_i \log_2 \left(\frac{1}{P_i} \right) \approx 0$$

For both very
common & very
rare events

$P_i \approx 1$

$P_i \approx 0$